# E9: Nernst and Donnan, and why cells need pumps

Week 14. Due Wed 14 Apr 23:59, reviewed Thu 15 Apr

About 2 hours. Theory: Donnan equilibrium and Membrane potentials and ion pumps. Use $k_BT/e = 26.7$ mV (310 K).

## 1. How much trapped charge can a cell afford?

Outside a cell: 140 mM Na , 10 mM K , 150 mM Cl . Inside, trapped macroions carry a charge density $\rho_m$ (in mM of elementary charges, negative).

a. Write a Python function that solves the Donnan equations for given outside concentrations and $\rho_m$, and returns the inside concentrations, the membrane potential and the osmotic excess inside. Check that $\rho_m/e = -125$ mM gives the result in the notes.
b. Plot $\Delta\mathcal{V}$ and the osmotic pressure $\Delta P$ against $-\rho_m/e$ from 0 to 200 mM.
c. A lipid membrane ruptures at a tension of about 10 mN/m. With the Laplace pressure $\Delta P = 2\gamma/R$, what pressure can a cell of radius 10 µm withstand? What is the largest trapped charge density it could have without pumping? Compare with the 125 mM of a real cell.
d. Show that for small $\rho_m$ the osmotic excess is $\Delta c \approx \rho_m^2/(4c_s)$, where $c_s = 150$ mM is the outside salt concentration. Why is it quadratic?

## 2. The pump–leak steady state

Use the squid axon data in the notes, and $g_{\mathrm{K}} = 25\, g_{\mathrm{Na}}$.

a. Derive the resting potential with a 3 Na : 2 K pump (notes) and calculate $\Delta\mathcal{V}$.
b. Some pumps exchange one ion for one ion and move no net charge. What would the resting potential be with a 1 Na : 1 K pump? Does the stoichiometry matter much for $\Delta\mathcal{V}$? What does it matter for?
c. Calculate the free energy needed for one cycle of the 3:2 pump at $\Delta\mathcal{V} = -60$ mV, in $k_BT$, and compare it with the 19 $k_BT$ from one ATP. What is the thermodynamic efficiency?
d. Why must chloride, which is not pumped in this model, have its Nernst potential equal to the resting potential?