E8: Cell volume, screening and why ions need channels

Week 13. Due Wed 7 Apr 23:59, reviewed Thu 8 Apr

About 2 hours. Theory: Processes at membranes. Use $k_B T$ at 310 K, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m.

1. Cell volume in a hypertonic solution

A cell has volume V_0 in a solution of osmolarity $c_0 = 300$ mM. A part V_b of its volume (proteins, membranes, organelles) does not take part in osmosis. The rest holds a fixed amount of solute that cannot cross the membrane, while water can.

a. The cell is moved to a solution of osmolarity c. Show that in equilibrium

$$V = V_b + (V_0 - V_b)\,\frac{c_0}{c}.$$

This is the Boyle–van 't Hoff relation.

b. A red blood cell has $V_0 = 90$ fL and $V_b \approx 0.4\,V_0$. What is its volume in a 600 mM solution? In a 150 mM solution? What is likely to happen in the second case?

c. How would you measure V_b in the lab? What should you plot to get a straight line?

2. How far do electric forces reach in a cell?

a. Calculate the Bjerrum length $\lambda_B = e^2/(4\pi\varepsilon k_B T)$ in water ($\varepsilon_r = 80$) and in the oily interior of a membrane ($\varepsilon_r = 2$).

b. Calculate the Debye length in 150 mM NaCl, in 1 mM NaCl and in pure water (H$^+$ and OH$^-$ at 10^{-7} M each).

c. What do the numbers mean for how proteins in the cytoplasm interact electrostatically?

3. How many ions make a membrane potential?

A cell membrane is a capacitor: two conducting salt solutions separated by a 4 nm thick insulating layer with $\varepsilon_r \approx 2$.

a. Calculate the capacitance per area and compare it with the measured value, about 1 µF/cm².

b. Using 1 µF/cm², how many ions per µm² must be moved across the membrane to create a potential of -70 mV? How many is that for a spherical cell of radius 10 µm?

c. Compare with the number of K$^+$ ions in the cell (140 mM). Is it a good approximation to treat the inside of the cell as electrically neutral?

4. Why ions do not cross lipid membranes

To leave water and enter the membrane, an ion has to give up the polarization of the water around it. The electrostatic self-energy of a sphere of radius r and charge q in a medium of permittivity ε is $E_{\text{self}} = q^2/(8\pi\varepsilon r)$ (the Born energy).

a. Calculate the difference in self-energy for K$^+$ ($r = 0.133$ nm) in the membrane ($\varepsilon_r = 2$) and in water ($\varepsilon_r = 80$), in units of $k_B T$.

b. Use the Boltzmann distribution to estimate the ratio of K$^+$ concentration inside the membrane to that in water (the partition coefficient). Compare with the partition coefficient of water in a lipid membrane, about 4×10^{-5}.