E5: Active Brownian particles and the Vicsek model

Week 9. Due Wed 10 Mar 23:59, reviewed Thu 11 Mar

About 2–3 hours. Theory: Active matter notes. Code: `src/vicsek/` (by Harish).

1. One active Brownian particle

Simulate 400 independent active Brownian particles with the finite-difference equations in the notes, using $v = 1$, $D_T = 0.1$, $D_R = 0.5$, $\Delta t = 0.01$ (arbitrary units) and 2×10^4 time steps. Plot the MSD against time on log–log axes.

a. Identify the diffusive, ballistic and enhanced-diffusive regimes. At what times do the crossovers happen? Compare with D_T/v^2 and $\tau_R = 1/D_R$.
b. Compare the long-time slope with $D_{\text{eff}} = D_T + v^2\tau_R/2$.

2. The flocking transition

Use `vicsek_model.generate_vicsek_data` (see `vicsek_model_exercise.py`) with $N = 400$, $L = 14$ (density ≈ 2), $r = 1$, $v = 0.1$ and 500 iterations.

a. For noise amplitudes $\eta = 0.1, 0.2, \ldots, 1.0$, compute the order parameter $\psi = \frac{1}{N}\left|\sum_i \mathbf{v}_i/v\right|$, averaged over the last 200 iterations. Plot ψ against η.
b. Where is the transition? Run once more at the η closest to the transition. How much does ψ fluctuate in time there, compared with far from it?
c. Why is $\psi \approx 1/\sqrt{N}$, not 0, in the disordered phase?