# E4: Drag, Reynolds number and the Langevin equation

Week 8. Due Wed 3 Mar 23:59, reviewed Thu 4 Mar

About 2 hours. Theory: Viscous drag and Theory of Brownian motion. Read Purcell's Life at low Reynolds number before Thursday 25 Feb.

## 1. Nelson

Nelson: Your Turn 5a, 5b and 5c.

## 2. Swimming *E. coli*

An *E. coli* bacterium (treat it as a sphere of radius $a = 1$ µm, density $\approx 10^3$ kg/m$^3$) swims at $v = 30$ µm/s in water.

a. Calculate its Reynolds number. Compare with you swimming (1 m, 1 m/s).
b. Calculate the Stokes drag force and the power needed to swim. Express the power in $k_BT$ per second.
c. The motor stops. Estimate how far the bacterium coasts. (Hint: the velocity relaxes with time constant $m/\xi$.)

## 3. The Langevin equation for your beads

a. For a 0.51 µm polystyrene bead (density 1050 kg/m$^3$) in water, calculate the relaxation time $m/(6\pi\eta r)$ in the solution of the Langevin equation. Compare it with the time between frames in the lab. Why can we drop the inertial term?
b. Show that the solution of the Langevin equation, with the transient dropped, gives $\langle x^2\rangle = 2Dt$ with $D = k_BT/(6\pi\eta r)$.
c. Calculate the expected $D$ for the beads at 20 °C and at 25 °C ($\eta = 1.00$ and 0.89 mPa · s). How precisely must you know the temperature to measure $k_B$ to 5 %?