E2: Random walks

Week 6. Due Wed 17 Feb 23:59, reviewed Thu 18 Feb

About 2 hours. Theory: Random walks.

1. A biased random walk

A walker in 1D takes n steps of length d: to the right with probability p, to the left with probability $1-p$. The steps are independent.

a. Show that $\langle x_n - x_0 \rangle = nd(2p-1)$.
b. Show that the variance is $\langle (x_n - x_0)^2 \rangle - \langle x_n - x_0 \rangle^2 = 4np(1-p)d^2$.
c. Check that you recover $\langle (x_n - x_0)^2 \rangle = nd^2$ for $p = 1/2$. What kind of motion does a bias correspond to physically?

2. From steps to a diffusion coefficient

A random walker takes a step of length d in a random direction every Δt. Show that in 1D this corresponds to $D = d^2/(2\Delta t)$. A 0.5 µm bead in water has $D \approx 0.9$ µm²/s. What step length corresponds to $\Delta t = 1/15$ s (one video frame)? Is the bead really taking steps of this size?

3. Simulation

Run `randomwalk_exercise.py`.

a. What slopes do you expect for the three MSD curves (in units of `stepsize`^{2})? Why?
b. Change `numpart` and `timesteps` so that the statistics are similar to your experiment (about 50 particles visible, 300–4500 frames). How large is the scatter of the fitted slope between runs?